

**AI-1526-M.E.-CV-19**  
**M.A./M.Sc. - (Previous)**  
**Term End Examination, Mar.-Apr.-2021**  
**MATHEMATICS**  
**Differential Geometry of Manifolds, Paper-IV**

**Time : Three Hours]**

**[Maximum Marks : 100**

**Note : Answer any Five Question. All Question carry equal marks.**

1. (a) Define differentiable manifold and show that the real projective space  $PR^n$  is differentiable manifold.  
(b) State and prove Local immersion theorem.
2. Let  $(M, g)$  be a Riemann manifold with sectional curvature  $k \geq k_0 > 0$ . Then show that for any geodesic  $c$  in  $M$ , the distance between two conjugate points along  $c$  is  $\leq \frac{\pi}{\sqrt{k_0}}$
3. (a) Show that the tangent bundle of a Lie group is trivial  $TG \cong G \times g$ .  
(b) Show that the range of the zero section of a vector bundle  $E \rightarrow M$  is a submanifold of  $E$ .
4. (a) State and prove Schur's Theorem.  
(b) Define the following with example:-
  - (i) Nijenhuis tensor
  - (ii) Conformal curvature tensor
  - (iii) Exterior derivative
  - (iv) Bundle homomorphism
5. Let  $g: L \rightarrow Q$  be a surjective submersion which is proper, show that  $g^{-1}(k)$  is compact in  $L$  for each compact  $k \subset Q$  and let  $Q$  be connected. Then show that  $(L, P, Q)$  is fibre bundle.
6. State and prove Generalized Gauss and Mainardi-Codazzi equations.
7. Show that the circle  $s' \subset \mathcal{C}$  is a Lie group under complex multiplication and the map
$$z = e^{i\theta} \rightarrow \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & I_{n-2} \end{bmatrix}$$
Is a Lie group homomorphism into  $SO(n)$ .
8. (a) Prove that every vector bundle of dimension  $n$  over  $V$  is associated to a principal bundle over  $V$  with group  $GL(n, R)$ .  
(b) Define the following:-
  - (i) Tangent bundle
  - (ii) Induced bundle
  - (iii) Principle fibre bundle
9. (a) Prove that Riemannian geodesic is Locally minimizing.  
(b) State and prove First variation formulae.
10. (a) Show that tangent bundle is a vector Bundle.  
(b) Prove that  $X^T = KMOTX$  for vector Bundle Homomorphism.